Global & Multicultural Perspectives  Symbolic Reasoning  Written Communication

The Honolulu Community College Foundations Board will review all proposals to ensure that approved courses meet Foundations Hallmarks. If clarification is needed, a Board member will contact you. If the Foundations Board and the General Education Committee approve the proposal, all sections of the course will be designated as satisfying the requirement. The course will be reviewed every five years.

1. Course information. Course Alpha MATH Course Number 203

If the course is cross listed, please provide the cross-listing: Alpha Number

Course Title: Calculus for Business and Social Sciences

2. Foundations area requested. Check one. Global & Multicultural Perspectives  Symbolic Reasoning  Written Communication

3. How many instructors currently teach this course? It makes a difference if there are only one or two instructors teaching this course versus ten instructors teaching this course. This question is asked to get an idea of how many instructors the department needs to communicate with to discuss this foundation course.

4. Syllabus. Submit a master syllabus. If multiple instructors teach the course and use varying texts and/or assignments, please include multiple representative syllabi for comparison. (Three is recommended.)

5. Hallmark Requirements. Provide an explanation of how each of the hallmarks for this proposed Foundation course will be satisfied. Try to completely answer how the course intends to meet each particular hallmark. Referencing assignments, tasks, and evaluations used in the course (as stated on the syllabus/syllabi being submitted) as supporting evidence would be very helpful.

6. Assessment. Provide a brief explanation of how the department will periodically review that this course has been meeting the Foundations Hallmarks including a description of what kinds of evidence will be collected to demonstrate this (Knowledge Survey results, sample of exam responses, writing samples, etc.). Also include a detailed description of how the department plans to have all instructors of this course share information with each other regarding how the hallmarks have been met. Please include a brief explanation of the assessment tools you will use to make this determination (such as Knowledge Surveys, Exams, Projects, Portfolios, etc.) and how you will use the results to make course improvements.

7. Signatures. The signatures of the initiator and the initiator’s Division Chair are required. The completed proposal must be routed to the Chair of the CPC before being delivered to the chair of the Foundations Board. No action on the part of the CPC is required unless the proposal also includes a new course Curriculum Action or a course modification Curriculum Action. The “routing” is a courtesy to the CPC. Signatures indicate approval/acceptance.
Official course description

Math 203  Calculus for Business and Social Sciences

Prerequisite: “C” or higher in Elementary Functions I (Math 135) or a HCC Placement Test recommendation of Math 140. A strong background in algebra is required.

Course Description

The course covers applications of calculus to business and economics including limits, derivatives, curve sketching, integration, and partial derivatives. These concepts and techniques are used to analyze and solve problems frequently encountered in business and social applications.

This course fulfills the Symbolic Reasoning requirement for the Foundation requirement for Honolulu Community College and the University of Hawaii at Manoa. See the Manoa General Education requirements.

Students will

- demonstrate an understanding of the beauty, power, clarity, and precision of formal systems through guided practice in calculus operations and problem-solving.
- demonstrate through performances on assessment exams, classwork, and homework exercises the concept of proof as a chain of inferences.
- apply formal rules of algorithms in calculus.
- demonstrate correct and effective use of the symbolic rules of calculus on assessment exams, classwork, homework exercises, or related projects.
- analyze rules and theorems to find the most effective solutions to problems.
- apply calculus principles to solve real-world problems related to management, finance, economics, and the social sciences.

Foundations Hallmarks – Symbolic Reasoning

1. Students will be exposed to the beauty, power, clarity, and precision of formal systems. How will the course meet this hallmark?

Math 203 is an applied calculus course with an emphasis on studying and solving problems arising in business management and economics. Although Math 203 is an applied course, a substantial amount of time will be spent on the mathematical theory, which will be introduced carefully and deliberately but in slightly less depth than in more theoretical calculus courses. For instance, the notion of limit is introduced via tables and graphs. Differentiation then is introduced as the limit of the difference quotient and integration as the limit of a Riemann sum. The abstract parts of the course provide students with the justification for the techniques that are used to analyze the properties of functions that commonly arise in business and economics applications. For example, the first derivative is used to find relative extrema while the second derivative is used to find concavity and these are used to graph polynomial, rational and radical functions. The applied part of the course uses these ideas and techniques to understand, study, and solve problems involving such topics as optimization and nonlinear supply and demand curves.
The beauty of formal systems is illustrated by discussing how the notion of finding the slope of a tangent line, a purely mathematical application led to solving optimization problems in business. The precision and clarity of formal systems can clearly be seen when one problem can be solved in a variety of ways.

Example: A firm has two plants. Suppose that the cost of producing $x$ units at plant X is given by $x^2 + 1000$ and the cost of producing $y$ units of the same product at plant Y is given by $3y^2 + 600$. If the firm has an order for 900 units, how many units should be produced at each of the plants to fill the order and minimize the cost of production? We see that the total cost $C = x^2 + 1000 + 3y^2 + 600$. Since the order is for 900 units, $x + y = 900$.

1) Writing $C$ in one variable, $C = (900 - y)^2 + 3y^2 + 1600$ or $4y^2 - 1800y + 811,600$. Algebraically, this problem can be solved by completing the square. $C = 4(y^2 - 450y + 50,625) - 4(50,625) + 811,600$ or $C = 4(y - 225)^2 + 609,100$. This represents a parabola (pointing upwards) with vertex $(225,609,100)$ which means $y$ is a minimum at $y = 225$. Solving for $x$, we get $x = 675$. Hence 675 of the items should be produced at plant X and 225 units should be produced at plant Y.

2) With the power of calculus, this same problem can be solved much faster, using the first derivative. The total cost is $C = 4y^2 - 1800y + 811,600$. Then $C' = 8y - 1800$. By setting the equation to 0, we get $y = 225$. Since $C'' = 8$ is positive, this indicates that 225 is a minimum value. Solving for $x$, we get $x = 675$.

3) Another method for solving this problem would be using Lagrange Multipliers with a constraint. $C = x^2 + 1000 + 3y^2 + 600$ and $x + y = 900$. We do not need to change the equation to one variable.

$$ F(x,y,\lambda) = x^2 + 1000 + 3y^2 + 600 - \lambda(x + y - 900). $$

$$ F_x = 2x - \lambda = 0 \quad F_y = 6y - \lambda = 0 \quad F_\lambda = 900 - x - y = 0 $$

Using the first two equations, $2x = 6y$ or $x = 3y$. Substituting this into the last equation, we get $900 = 3y + y$ or $4y = 900$. $y = 225$, and $x = 675$.

Seeing how such problems can be rigorously analyzed and solved using calculus, students will realize the tremendous advance that calculus represents over algebraic techniques.

2. Instructors will help students understand the concept of proof as a chain of inferences. How will instructors help students understand this concept?

The proofs of important theorems will be discussed in class. The essential limit, differentiation and integration formulas will be derived in class as well. Students will similarly be responsible for some proofs and derivations, though not as many as in a more theoretical calculus course. Inductive reasoning requires searching for some pattern among a finite number of examples, while deductive reasoning reaches a general conclusion from definitions and general rules that have already been proved.

Example: Suppose $f(x) = x^2$, find $f'(x)$. Students would solve this using the definition of the derivative:

$$ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x + h)}{h} = 2x. $$

Next, suppose $f(x) = x^3$, find $f'(x)$
\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2. \]

Using a few more similar examples, we inductively conclude if \( f(x) = x^n \), then \( f'(x) = nx^{n-1} \).

A formal proof would then follow:

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots + h^n - x^n}{h} \]

\[ = \lim_{h \to 0} \frac{h(nx^{n-1} + 2\frac{n(n-1)}{2}x^{n-2}h + \ldots h^{n-1})}{h} = nx^{n-1}. \]

In other areas, counterexamples will be used to help students see the importance of careful reasoning and in their understanding of implications and quantifiers in the theorems and techniques.

3. **Instructors will teach students how to apply formal rules of algorithms. How will instructors meet this hallmark?**

Calculus includes a number of rules and algorithms for solving a wide variety of problems involving limits, rates of change, and optimization. The formal rules for evaluating limits, differentiation, and integration are central to the course and students will be required to learn, select and use them appropriately. Students are also required to select and apply appropriate calculus techniques to solve concrete application problems.

*Example:* The demand for a car waxing is given by \( x = 2400 - 75p \), where the current price is $15 a car. Can revenue be increased by lowering the price and thus attracting more customers?

Intuitively, one would reason that a lower price should attract many new customers and increase revenue.

A student can elect to solve this problem by writing \( R(p) = p(2400 - 75p) \) and use the properties of this function to answer the question. Finding the first derivative and making a sign graph, we get: \( R'(p) = 2400 - 150p. \)

\[ \frac{dx}{dp} = \frac{p}{2400 - 75p} \]

which shows that until \( p = 16 \), the Revenue function is increasing, hence at \( p = 15 \), by lowering the price, the revenue would not increase, but decrease.

Or he could elect to solve this problem using elasticity of demand which is the ratio of the percent change of a quantity demanded divided by the percent change in its price \( p \).

\[ \eta = \frac{\frac{dx}{dp}}{\frac{p}{2400 - 75p}} = \frac{p}{2400 - 75p} \cdot 75. \] AT \( p = 15 \), \( \eta = -\frac{15}{17} \). Since \( |\eta| < 1 \), the demand is inelastic; hence revenue will decrease when the price decreases.

4. **Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence. What symbolic techniques will be required and in what contexts? How will presentations and evaluations of evidence be incorporated into the course?**
In the course of solving concrete problems, students must select an appropriate mathematical model of the situation, manipulate the model by applying appropriate techniques, interpret the results of the manipulations, and check the correctness of their results.

*Example:* When the price of a certain item is \( p \) dollars per unit, customers demand \( x \) hundred units of the item where \( x^2 + 3px + p^2 = 63 \). If price is decreasing at a rate of 40 cents per month, how fast is the demand \( x \) changing with respect to time when the price is $5?

A student needs to rewrite this problem using appropriate symbols. He would write:

Given: \( x^2 + 3px + p^2 = 63 \) \( p = 6 \) \( \frac{dp}{dt} = -.40 \). Find \( \frac{dx}{dt} \).

First find \( x \) when \( p = 6 \), \( x = 3 \). Then differentiating with respect to \( t \), we get

\[
2x \frac{dx}{dt} + 3p \left( \frac{dx}{dt} \right) + 3x \left( \frac{dp}{dx} \right) + 2p \left( \frac{dp}{dx} \right) = 0.
\]

Solving for \( \frac{dx}{dt} \) and substituting the given data, we get \( \frac{dx}{dt} = .35 \). Interpreting this result, when the price is $5, and is decreasing at the rate of 30 cents a month, the demand is increasing about 35 units per month. This can be checked reducing the price in the next month by 40 cents and getting $5.60. Plugging this into the modeled equation, we get \( x^2 + 3(5.6) + (5.6)^2 = 63 \) and solving for \( x \). We get \( x = 34.8311 \) which checks with our answer.

5. The course will include computational and/or quantitative skills. What reasoning skills will be taught in the course? What computational and/or quantitative skills will be taught in the course? How will you assess this and provide evidence that students are meeting this hallmark?

Although considered an applied mathematical course, Math 203 still requires an understanding of calculus concepts. As such, proof and derivations play an important role in the course. At the same time, since Math 203 is an applied course, many of the problems will involve concrete situations in which students must apply reasoning process in more than one stage of the problem’s solution. For example, a student must use reasoning to select an appropriate mathematical model and may have to use reasoning again at the end of the problem to check the reasonableness of the “answer”.

*Example:* Suppose an agency charges $10 per person for admission to a concert if there are 30 people in a group. For every person above 30, the agency reduces the ticket by $.20. How many people will maximize revenue for the agency? A student needs to “reason” that the more people, the more revenue BUT at the same time, too many people would lesson revenue as the admission price is dropping. He also needs to reason that at most, 50 people can be a group as otherwise, the ticket price would be negative (they would be paying people to go to the concert). It is by reasoning that the revenue function can be set up:

\[
R(x) = (\text{number of tickets})(\text{price per ticket}).
\]

For each new person above 30, the ticket price is 20 cents less. So if \( x \) represents the number of people above 30, the revenue function can be written as \( R(x) = (30 + x)(10 - .20x) \). Since we do not want a group larger than 50, \( x \in [0, 20] \). After solving with \( x = 10 \), it seems reasonable that the halfway point would be where revenue would be maximized. Though computations do play a substantial role (as they do in virtually any mathematics course), mastering computational skills alone is insufficient to learn the required material.
6. Instructors will build a bridge from theory to practice and show students how to traverse this bridge. How will instructors help students make connections between theory and practice?

The two major topics of the course are differentiation and integration. In each of these topics, general principles are first discussed on an abstract level. Next they are discussed on a practical level in the solution of real world problems.

Example:

1) Find the area of a region bounded by the curve \( y = f(x) \), from \( x = a \) to \( x = b \). This problem is first estimated using rectangles and increasing the number of rectangles to get a better estimation. By using limits, we get

\[
A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \left( \frac{b - a}{n} \right)
\]

and finally

\[
A = \int_{a}^{b} f(x) \, dx.
\]

2) This is then used to solve problems such as “Find the area of the region bounded by \( f(x) = x^2 + 4x + 2 \) from \( x = 0 \) to \( x = 5 \).

\[
A = \int_{0}^{5} (x^2 + 4x + 2) \, dx = \frac{305}{3}
\]

3) Finally the integral is used in business problems such as “A store finds that its sales change at a rate given by \( S'(t) = -3t^2 + 250t \) and \( 0 \leq t \leq 20 \) where \( t \) is the number of days after an advertising campaign ends. Find the total sales for the first week after the campaign ends.

\[
\text{Total sales} = \int_{0}^{7} (-3t^2 + 250t) \, dt = 5782.
\]

Assessment

Any faculty member assigned to teach Math 203 must become familiar with both the course content and the FS hallmarks. The learning outcomes and FS hallmarks will be assessed by the use of embedded questions on exams. It is the responsibility of the math department liaison to meet with the instructor(s) and ensure that the course learning outcomes and the FS hallmarks are being satisfied. Math 203 faculty will meet, beginning with the spring 2011 semester to correlate the learning outcomes with the appropriate hallmark and to specify which questions to embed and determine whether the FS hallmarks are being addressed via the specific questions chosen. After the initial assessment, the Math 203 faculty will meet each year for a two year period to determine whether the FS hallmarks are being addressed. After the initial assessment, the Math Faculty will meet every three years for a review of the process.

Syllabus

See attachment.
Math 203  Calculus for Business and Social Sciences  Spring, 2009

Instructor: Faye Tamakawa  e-mail: ftamakaw@hawaii.edu
Office:  7-413  Office Phone:  845-9443

Office Hours:  MW  10:30-11:30  Other hours by appointment only
TTh  11:45-12:45

Course description: The course covers application of calculus to business and economics including limits, derivatives, curve sketching, integration and partial derivatives. This course fulfills the Symbolic Reasoning requirement for the Foundation requirement for Honolulu Community College and the University of Hawaii at Manoa.

Symbolic Reasoning Objectives:

Students will
- Demonstrate an understanding of the beauty, power, clarity, and precision of formal systems through guided practice in problem solving involving trigonometry and analysis of conic sections.
- Demonstrate through performances on assessment exams, classwork, and homework exercises the concept of proof as a chain of inferences.
- Apply formal rules of algorithms in trigonometry and algebraic representations of conic sections.
- Demonstrate correct and effective use of the symbolic rules of trigonometry, conic functions, and exponential and logarithmic functions on assessment exams, classwork, homework exercises or related projects.
- Analyze rules and theorems to find the most effective solutions to problems.
- Apply trigonometric and conical analysis principles to solve real-world problems related to real-world problems.

Student Learning Outcomes:

At the end of the course, the student will be able to:
- calculate derivatives and partial derivatives of non-trigonometric elementary functions and their sums, products, quotients and compositions
- use calculus to perform optimization methods
- evaluate definite and indefinite integrals by using basic formulas and substitution
- apply definite and indefinite integrals to business/economics problems
- know the basics about functions of more than one variable
- solve applied problems by using functions of more than one variable

Prerequisite: “C” or higher in Elementary Functions I (Math 135) or a HCC Placement Test recommendation of Math 140. A strong background in algebra is required.

Required supplies: 1) scientific calculator       2) 3 ring binder (recommended)

Method of Instruction: Most class sessions will include a brief review of concepts and/or student-initiated questions regarding homework, and a lecture on new material. Students will be called on periodically throughout the lecture to participate in any given problem. If you are experiencing difficulty in understanding any concept, see me during office hours.

Remember - Math is NOT a spectator sport. You learn by doing, not watching others do the work.

Testing and Grading: Your grade will be determined by scores earned on 3 unit exams, daily quizzes, and a final exam. Final grades will be determined by the following scale:

- A = 90% - 100%
- B = 80% - 89%
- C = 70% - 79%
- D = 60% - 69%
- W = Withdrawal
- I = Incomplete
- F = Below 60%

If you are not attaining a satisfactory grade, you need to withdraw by the deadline (March 20, 2009) as listed in the schedule of courses. Any student who “disappears” without officially withdrawing will receive an "F".

Policies on "Incomplete" grades:
Incomplete grades will be given only to students who are achieving passing grades and are very close to completing the course. In addition, the student must have a certified medical reason, a death in the family or any other extreme circumstance to qualify for this grade. The work must be made up in the fall or the grade will revert to an “F”.

Instructor Policies:
♦ For all exams and quizzes, you must show all your work. Do not skip steps. Remember NO WORK means NO CREDIT! I grade your work, not just the answer.

♦ Quizzes will be given each class period, consisting of problems similar to the problems discussed in class. All quizzes are “take home” quizzes. These quizzes should be done neatly on another piece of paper. All answers must be circled. Skip a line between each problem. Leave room for comments. All quizzes are due at the beginning of class. After the due date, no quiz will be accepted. No quiz should be done in class. If I see you doing a quiz in class, you will forfeit that quiz. You may use your notes, book, etc to do the quiz. You are not allowed to ask me for help on any problem on the quiz, but I will be happy to explain a problem similar to the one on the quiz if you ask me. All quizzes should be done alone – do not let another student copy your quiz. If you are absent, you may drop off your quiz before class begins (even the day before). Do not have another student turn in your quiz – it is your responsibility to turn in your own quiz. Each quiz counts 10 points and will be averaged for each unit. At the end of the semester, the total will be 100 points, which is equivalent to one exam.

♦ If you are absent on the day of the exam you MUST call to explain your absence. If I am not in, leave your message on the answering service. You will be allowed to make up this exam in the Testing Room (picture ID required) on the third floor. You do not need confirmation after you call. Just go to the Testing room when you return. If you do not call, you will be allowed to make up
the exam, but 2 pts will be deducted from the exam for not calling in. The exam should be made up within one week. For the first exam taken late, no penalty will be given. However, any subsequent exam taken late will be penalized 10 points.

✦ Attendance is taken each class period. Attendance does not count on your grade but is required as we need to state the last day a student attended class, should they not pass the course.

✦ If you have a documented disability which requires special accommodations to fulfill the course requirements, please see me.