Honolulu Community College
University of Hawai‘i
General Education
Foundations Course Designation Proposal Form
For Fall 2009 – Summer 2014

Global & Multicultural Perspectives  Symbolic Reasoning  Written Communication

The Honolulu Community College Foundations Board will review all proposals to ensure that approved courses meet Foundations Hallmarks. If clarification is needed, a Board member will contact you. If the Foundations Board and the General Education Committee approve the proposal, all sections of the course will be designated as satisfying the requirement. The course will be reviewed every five years.

1. Course information.
   Course Alpha MATH
   Course Title: Survey of Mathematics
   If the course is cross listed, please provide the cross-listing: Alpha Number

2. Foundations area requested.
   Check one.
   Global & Multicultural Perspectives ☐  Symbolic Reasoning ☒  Written Communication ☐

3. How many instructors currently teach this course? It makes a difference if there are only one or two instructors teaching this course versus ten instructors teaching this course. This question is asked to get an idea of how many instructors the department needs to communicate with to discuss this foundation course.

4. Syllabus. Submit a master syllabus. If multiple instructors teach the course and use varying texts and/or assignments, please include multiple representative syllabi for comparison. (Three is recommended.)

5. Hallmark Requirements. Provide an explanation of how each of the hallmarks for this proposed Foundation course will be satisfied. Try to completely answer how the course intends to meet each particular hallmark. Referencing assignments, tasks, and evaluations used in the course (as stated on the syllabus/syllabi being submitted) as supporting evidence would be very helpful. See the previously submitted Religion 150 application for examples located at http://honolulu.hawai.ledu/intranet/articulation/foundations/REL150.pdf

6. Assessment. Provide a brief explanation of how the department will periodically review that this course has been meeting the Foundations Hallmarks including a description of what kinds of evidence will be collected to demonstrate this (Knowledge Survey results, sample of exam responses, writing samples, etc.). Also include a detailed description of how the department plans to have all instructors of this course share information with each other regarding how the hallmarks have been met. Please include a brief explanation of the assessment tools you will use to make this determination (such as Knowledge Surveys, Exams, Projects, Portfolios, etc.) and how you will use the results to make course improvements.

7. Signatures. The signatures of the initiator and the initiator’s Division Chair are required. The completed proposal must be routed to the Chair of the CPC before being delivered to the chair of the Foundations Board. No action on the part of the CPC is required unless the proposal also includes a new course Curriculum Action or a course modification Curriculum Action. The “routing” is a courtesy to the CPC. Signatures indicate approval/acceptance.

Initiated by: ____________________________
   Mike Kaczmarski
   Initiator’s printed name
   Date

Approved by: ____________________________
   Kerry Tanimoto
   Division Chair’s printed name
   Date

Routed via: ____________________________
   MARCIA ROBERTS-DEUTSCH
   CPC Chair’s printed name
   Date

Accepted by: ____________________________
   Foundation Board Chair’s printed name
   Date
Official Course Description

MATH 100 – Survey of Mathematics

Course Description

- Prerequisite: C or higher in Math 25 or placement in Math 100
- Recommended Prep: Placement in ENG 22/60

A general survey of mathematics with emphasis on its historical development and the role it plays in modern society.

Student Learning Outcomes

Upon successful completion of Math 100, students will be able to:

1. Perform inductive reasoning by searching for patterns among specific examples and forming general conjectures.
2. Find counter-examples to inductive reasoning general conjectures.
3. Perform deductive reasoning by using previously established general principles.
4. Perform set operations of union, intersection, and complements by roster or Venn diagrams.
5. Develop set laws by inductive or deductive reasoning.
6. Solve applied problems (e.g., survey analysis) using set operations.
7. Perform operations of conjunction, disjunction, and negation on logic statements that are either represented symbolically or by Euler diagrams.
8. Construct truth tables for compound logic statements.
9. Develop logic laws by inductive or deductive reasoning.
10. Determine the validity of logical arguments by truth tables, standard arguments (e.g., the Law of Contraposition, the Fallacy of the Inverse), or Euler circles.
11. Represent numbers using various numeration systems, both ancient and modern.
12. Perform arithmetic operations in various numeration systems.
13. Apply the binary, octal, and hexadecimal numeration systems to the modern digital world.
14. Count the elements of finite sets by systematic listings, trees, permutations, combinations, or the Fundamental Counting Principle.
15. Develop counting formulas by inductive or deductive reasoning.
17. Determine expected values of real-life events in games.
18. Summarize real-life data sets by bar graphs or histograms and interpret such summaries.
19. Determine means, medians, modes, ranges, and standard deviations of real-life data sets.
20. Solve applied problems (e.g., grade determination, stock volatility) by using descriptive statistics.

In general, the course will develop the student’s quantitative and analytical reasoning abilities and will familiarize the student with some of the different areas of mathematics so that the student might gain a better understanding of and appreciation for mathematics. Completion of
this course with a “C” grade or higher satisfies the three credits of quantitative reasoning requirement of many University of Hawaii Manoa Programs.

**Assessment**

Each hallmark is covered by student learning outcomes.

- **Hallmark 1:** SLOs 5, 6, 9, 10, 13, 15, 16, 17
- **Hallmark 2:** SLOs 1, 2, 3, 5, 9, 15
- **Hallmark 3:** SLOs 6, 10, 13, 14, 16, 17, 20
- **Hallmark 4:** SLOs All 1 through 20
- **Hallmark 5:** SLOs 1, 2, 3, 5, 9, 15
- **Hallmark 6:** SLOs 6, 10, 13, 16, 17, 20

Since the SLOs are covered, then the Hallmarks will also be covered.

The mathematics department will assign a liaison(s) for Math 100. The liaison(s) will discuss with all Math 100 instructors how the hallmarks relate to SLOs and the need to cover and assess SLOs. Each instructor will choose questions pertinent to each SLO, assign each question a point value, and embed them in homework, quizzes, tests, or the final exam. The liaison(s) will check to see that suitable questions have been embedded.

To assess student performance on an SLO, the point value of all the embedded questions pertinent to the SLO will be totaled. A student that has at least 70% of the total will be said to have achieved competence on the SLO. Each semester, for each Math 100 class, the instructor will tabulate how many students have achieved competence on each SLO, and the tabulations will be discussed with the liaison(s). The instructor and liaison(s) may agree to modify the way the instructor covers an SLO.

The liaison(s) will keep a copy of each Math 100 instructor’s SLO student performance tabulation. Every 3 to 5 years the liaison(s) will summarize all collected tabulations and discuss the summary at a meeting of the mathematics department. Strategies in the ways that SLOs can be covered, or whether or not an SLO should be modified, will be discussed.

**Foundations Hallmarks – Symbolic Reasoning**

1. *Students will be exposed to the beauty, power clarity, and precision of formal systems. How will the course meet this hallmark?*

The topics are discussed from two complementary viewpoints: theoretical and practical. The theoretical viewpoint infers general laws expressed in precise symbols via inductive or deductive reasoning. The practical viewpoint then applies the general laws to the solution of real-world problems, often resulting in a numerical result whose relevance is interpreted by the student. For example, in Set Theory, the Inclusion-Exclusion Principle for Finite Sets, \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \), can be proved using a Venn diagram. The Principle can then be used to analyze a survey.
Part of the beauty and power of formal systems is that different systems often have remarkably similar general laws. Two examples follow: 1) In Probability Theory, the probability of two events, E and F, is calculated by \( P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) \), which can easily be deduced from the Set Theory Principle. The Probability Inclusion-Exclusion Principle can be used to calculate the expected value of a life insurance policy. 2) Using inductive reasoning, it can be deduced that \( 2^n \) can be used in Set Theory to calculate the number of subsets of a finite set and in Logic to calculate the number of lines required in a truth table. To help explain why this happens, students are asked to consider the common bi-polar nature of both sets and logical statements. An object is either inside a set or outside a set; a logical statement is either true or false.

Another part of the beauty and power of formal systems is that general laws can be proved many different ways. For example, in Logic, the Law of Disjunctive Syllogism can be proved deductively by using a truth table, a Venn diagram, or a chain of previously proved laws.

2. Instructors will help students understand the concept of proof as a chain of inferences. How will the instructors help students understand this concept?

The course begins with a discussion of inductive and deductive reasoning. Inductive reasoning requires searching for a pattern among a finite number of several specific examples and then using the pattern to infer a general conclusion. Students will learn that while inductive reasoning often leads to correct conclusions, it may sometimes lead to an incorrect conclusion. They will search for counterexamples to show that a conclusion is incorrect, but they will also learn that finding counterexamples is not always easy.

Thus the need for deductive reasoning, where a general conclusion is inferred from general assumptions or general rules that have been previously been proved. Two examples follow:

1) In Logic, using either truth tables or Venn diagrams, it can be proved that \( \sim(\sim p) = p \), \( \sim(p \lor q) = p \land \sim q \), and \( p \rightarrow q = \sim p \lor q \). These rules can next be precisely chained together to deductively infer the Conjunctive Form of the Negation of a Conditional Statement, \( \sim(p \rightarrow q) = \sim(\sim p \lor q) = \sim(\sim p) \land \sim q = p \land \sim q \).

2) In Probability Theory, it can first be proved that \( P(\text{impossible event}) = 0 \), \( P(\text{certain event}) = 1 \), and \( P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) \), the Inclusion-Exclusion Principle. These rules can next be precisely chained together to deductively infer the exact relationship between \( P(E) \) and \( P(\sim E) \). \( P(E) + P(\sim E) = P(E) + P(\sim E) - 0 = P(E) + P(\sim E) - P(\text{impossible event}) = P(E) + P(\sim E) - P(E \text{ or } \sim E) = P(E \text{ or } \sim E) = P(\text{certain event}) = 1 \). Students will also learn that deductive reasoning is not always easy.

3. Instructors will teach students how to apply formal rules of algorithms. How will instructors meet this hallmark?

The topics of Sets, Logic, Numeration Systems, Counting, Probability, and Descriptive Statistics each have their formal laws and principles. In each of these topics, students will have to learn, select, and correctly apply them, especially in the solution of real-world problems. For example, suppose a screening committee of 4 men and 3 women is to be formed from a pool of 10 men
and 8 women, and a student is asked how many different committees can be formed. The student must first realize that the multiplicative Counting Principle applies, since forming the committee is a two-stage process of first selecting the men and then selecting the women. Then the student must realize that order is not important, so the number of ways each stage can be accomplished can be calculated by the use of combinations. Then the student must correctly apply the combination formula and calculate the results. The student would need to show work as follows:

Number of different committees = \( C_{10,4} \times C_{8,3} = 21 \times 56 = 11,760 \).

4. **Students will be required to use appropriate symbolic techniques in the context of problem solving, and in the presentation and critical evaluation of evidence. What symbolic technique will be required and in what context? How will presentations and evaluation of evidence be incorporated into the course?**

The topics of Sets, Logic, Numeration Systems, Counting and Probability each have their own symbolism. In each topic, students will be required to express problems symbolically, manipulate the symbols using various laws or principles, arrive at a symbolic conclusion, and determine the meaning and reasonableness of the symbolic conclusion. Two examples follow:

1) In Logic, a student can be required to explain how the contract “If you score 85% on the final exam, then I will give you a C for the course” can be broken. With \( s \) representing “you score 85% on the final exam,” and \( g \) representing “I will give you a C for the course,” the student would represent breaking the contract as a negation of a conditional statement, apply a formal rule that allows the student to rewrite the contract in an equivalent conjunctive form, and then state its final meaning: \( \neg(s \rightarrow g) = s \land \neg g \), which means “you score 85% on the final exam, but I don’t give you a C;”

2) In Numeration Systems, using only base sixteen numerals and base sixteen algorithms of subtraction with borrow, a student can calculate that \( 103_{sixteen} - D6_{sixteen} = 2D_{sixteen} \). The reasonableness of the answer can be ascertained by converting all base sixteen numerals to base ten numerals. Indeed \( 103_{sixteen} = 259_{ten}, \ D6_{sixteen} = 214_{ten}, \) and \( 2D_{sixteen} = 45_{ten} \), which checks.

5. **The course will not focus solely on computational skills. What reasoning skills will be taught in the course?**

The students will learn how to draw general conclusions using either inductive or deductive reasoning along with the advantages and disadvantages of each and the need for both. For example, In 1742, Christian Golbach applied inductive reasoning to conjecture that “Every even number greater than 2 can be written as the sum of two prime numbers.” Many have tried to disprove the conjecture by finding a counterexample, even employing supercomputers to look at billions of specific examples, but no counterexample has yet been found. Many others have tried to prove the conjecture using deductive reasoning, but no one has yet found such a proof either.

When a basic principle is difficult to prove with deductive reasoning, students will understand its reasonableness via inductive reasoning. For example, an examination of three or four specific examples convinces most students that the general expression \( 2^n \) can be used to count the number of subsets of a finite set and determine the number of lines needed in a truth table. When a basic principle is easy to prove with deductive reasoning, a deductive proof will be given. For example, De Morgan’s Laws for statements can be proven using a general truth table.
Students will also learn how to chain previously proved laws to deduce new laws. For example, in Login, once the Disjunctive Form of a Conditional Statement and the Law of Detachment have been proved, they can be chained to deduce the Law of Disjunctive Syllogism. [The Law of Syllogism can also be proved independently using either a truth table or a Venn diagram.]

Student will always be asked to think about the reasonableness of an answer, in the context of what they have learned. If the answer is expressed in words, does it sound about right? If the answer is numerical, is it in the ball park? Is I possible to get a totally different answer if the problem is done by another method?

6. Instructors will build a bridge from theory to practice and show students how to transverse this bridge. How will instructors help students make connections between theory and practice?

Some of the major topics of the course are Sets, Logic, Numeration Systems, Counting, and Probability. In each of these topics, general principles are first discussed on an abstract level. Then they are discussed on a practical level in the solution of real-world problems. Some examples follow: 1) In Sets, many set laws are proved using general Venn diagrams, where the regions are given general names. But the Venn diagrams can be made more specific, with specific numbers placed in each region to indicate the number of elements in that region. Using Venn diagrams in this specific manner allows one to analyze real-world surveys. 2) In Logic, a student may use the contrapositive of a conditional statement to restate a contract in equivalent language. 3) In Numeration Systems, using the binary representation of numbers and the simple rules of binary arithmetic, one can understand how digital computers use low voltage, (symbolized by 0) and high voltage, (symbolized by 1) to perform arithmetic computations on numbers. 4) In Counting and Probability, general formulas can be applied to calculate the probability of a specific outcome in a game of chance (such as being dealt a hand of two aces and two kings in a game of 5-card poker) or the expectation of buying a single raffle ticket.

Syllabus:

See attachment.
MATH 100 Survey of Mathematics
Fall, 2009

Instructor: Carol Hiraoka
Office: 7-418; Phone: 845-9405
E-mail: carolh@hcc.hawaii.edu
Office Hours: Mon through Thurs: 11:30 am – 1:00 pm
Mon & Wed: 4:30 pm - 5:15 pm
Other hours available by appointment

COURSE DESCRIPTION: A general survey of mathematics, with emphasis on its historical
development and the role it plays in modern society. (3 credits)

PREREQUISITES/COMMENTS: An entering student must have received a “C” or higher in
MATH 25 (Elementary Algebra II) or have placed in Math 100. Recommended preparation:
Placement in ENG 22/60.

The student should have already acquired the skills of elementary algebra including such topics
as polynomial fractions, linear functions and equations, quadratic functions and equations,
irrational numbers, radical expressions, and number systems.

Symbolic Reasoning Objectives:

Students will
• Demonstrate an understanding of the beauty, power, clarity, and precision of formal
  systems through guided practice in problem solving via inductive or deductive reasoning,
  on the theoretical level, and applying general laws to real-world problems on the practical
  level.
• Demonstrate through performances on assessment exams, classwork, and homework
  exercises the concept of proof as a chain of inferences, using both inductive and
  deductive reasoning.
• Learn, select and correctly apply formal rules of algorithms in selected real-world
  applications in areas of Sets, Logic, Numeration Systems, Counting and Probability, and
  Descriptive Statistics.
• Demonstrate correct and effective use of the symbolic techniques of algebra on
  assessment exams, classwork, homework exercises or related projects, particularly in
  topics of Sets, Logic, Numeration Systems, Counting and Probability, and Descriptive
  Statistics.
• Analyze rules and theorems to find the most effective solutions to problems.
• Apply algebraic principles to solve real-world problems related to real-world problems.

Student Learning Outcomes

Upon successful completion of Math 100, the student will be able to:
• Perform inductive reasoning by searching for patterns among specific examples and
  forming general conjectures.
• Find counter-examples to inductive reasoning general conjectures.
• Perform deductive reasoning by using previously established general principles.
• Perform set operations of union, intersection, and complements by roster or Venn diagrams.
• Develop set laws by inductive or deductive reasoning.
• Solve applied problems (e.g., survey analysis) using set operations.
• Perform operations of conjunction, disjunction, and negation on logic statements that are either represented symbolically or by Euler diagrams.
• Construct truth tables for compound logic statements.
• Develop logic laws by inductive or deductive reasoning.
• Determine the validity logical arguments by truth tables, standard arguments (e.g., the Law of Contraposition, the Fallacy of the Inverse), or Euler circles.
• Represent numbers using various numeration systems, both ancient and modern.
• Perform arithmetic operations in various numeration systems.
• Apply the binary, octal, and hexadecimal numeration systems to the modern digital world.
• Count the elements of finite sets by systematic listings, trees, permutations, combinations, or the Fundamental Counting Principle.
• Develop counting formulas by inductive or deductive reasoning.
• Determine probabilities of real-life events in sample spaces.
• Determine expected values of real-life events in games.
• Summarize real-life data sets by bar graphs or histograms, and interpret such summaries.
• Determine means, modes, medians, ranges, and standard deviations of real-life data sets.
• Solve applied problems (e.g., grade determination, stock volatility) by using descriptive statistics.

In general, the course will develop the student’s quantitative and analytical reasoning abilities and will familiarize the student with some of the different areas of mathematics so that the student might gain a better understanding of and appreciation for mathematics. Completion of this course with a “C” grade or higher, satisfies the three credits of quantitative reasoning requirement of many University of Hawaii Manoa programs.

**TEXT AND REFERENCES:** “Mathematics All Around” by Thomas L. Pirnot, 4th Edition, with use of MyMathLab. (Homework will be posted and completed via MyMathLab access on the internet.)

**EQUIPMENT AND MATERIALS:** The student is required to purchase a scientific calculator.

**EXAMS:** There will be three exams, one project, and a final exam. The final exam will cover the last topics since the third exam. Each exam and the project will have equal weight (one fifth each).

Exam 1:  
Chapter 1: Problem Solving  
Chapter 2: Set Theory

Exam 2:  
Chapter 3: Logic  
Chapter 4: Graphs
Exam 3:  Chapter 13: Counting  
Chapter 14: Probability  
Chapter 15: Descriptive Statistics  
Final:  Chapter 5: Numerations Systems  
Chapter 9: Consumer Mathematics  
Chapter 11: Apportionment  
Chapter 12: Voting  

**GRADING:**  
A = 90%  
B = 80%  
C = 70%  
D = 60%  
F below 60%  

“**N” Grade Policy:**  No “N” grade will be assigned for this class. Withdrawals without grade penalty must be done before the prescribed deadline (October 10, 2009).

**Test Makeup Policy:** Under special arrangements, missed exams can be made up. The make up exam must be taken before the corrected exam is returned to the class, so you should plan to take any make up the day following the original test. You must call the above telephone number during office hours on the day of the missed test to schedule a time and place for the make up.

**ATTENDANCE & HOMEWORK:**  
*You are expected to attend each class and do the assigned homework.*  
Attendance will be taken for administrative purposes. Homework is posted online on MyMathLab and must be completed online. Homework will not be collected, but completion will be verified by the instructor online.

**CELL PHONE POLICY:** All cell phones should be turned off or set in silent mode. Do not make or accept phone calls during class time.
1. **Perform inductive reasoning by searching for patterns among specific examples and forming general conjectures.**

   i) When using specific examples to form a conjecture, you are using:
   a) Deductive reasoning, b) Inductive reasoning, c) Educated guessing, d) none of the above.

   ii) List the next two numbers of the sequence: 1, 4, 9, 16, 25, 36,…

   iii) Determine the units digit of \(3^{31}\).

   iv) Given \(4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, \text{and} 10 = 3 + 7\). Therefore, any even natural number can be written as the sum of two primes. ____________ reasoning?

2. **Find counter-examples to inductive reasoning general conjectures.**

   i) A math student reasons as follows: "Consider the value of the polynomial \(n^2 – n + 41\) for different natural numbers, \(n\). For \(n = 1\), \(1^2 – 1 + 41 = 41\), which is prime. For \(n = 2\), \(2^2 – 2 + 41 = 43\), which is prime. For \(n = 3\), \(3^2 – 3 + 41 = 47\), which is prime. For \(n = 4\), \(4^2 – 4 + 41 = 53\), which is prime. Therefore, if \(n\) is any natural number, the value of \(n^2 – n + 41\) will be prime.” Supply a counter-example which shows that this reasoning is incorrect. Be sure to show enough work to justify your counter-example. A list of the first 100 primes is provided.

3. **Perform deductive reasoning by using previously established general principles.**

   i) When reaching a conclusion by applying general principles, you are using:
   a) Deductive reasoning, b) Inductive reasoning, c) Educated guessing, d) none of the above.

   ii) \((x + y)^2 = (x + y)(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2\). Therefore, \((x + y)^2 = x^2 + 2xy + y^2\). This is an example of _______________ reasoning.

   iii) Let A and B be any sets. When Venn diagrams for \((A \cup B)’\) and \(A’ \cap B’\) are correctly shaded in, the resulting diagrams are the same. Therefore, \((A \cup B)’ = A’ \cap B’\). This is an example of _______________ reasoning.

   iv) Draw a Venn diagram for \(A \cup (B \cap C)\).

   v) Use previously established set laws to simplify \((A’ \cup B’)’\). What type of reasoning was used?

4. **Perform set operations of union, intersection, and complements by roster or Venn diagrams.**

   i) List the elements of the following sets:
   \[U = \{1, 2, 3, 4, 5, 6, 7\}, \quad A = \{3, 4, 5, 6\}, \quad B = \{1, 2, 3, 5\}\] find:
   a) \(A’\), b) \(A \cap B\), c) \(A \cup B\), d) \(A – B\)

   ii) List the elements of the following sets:
   \[U = \{15, 16, 17, 18, 19, 20, 21, 22, 23\}, \quad D = \{16, 19, 21\}, \quad E = \{16, 18, 19, 20\}, \text{and} \quad F = \{15, 17, 18, 19, 21\}\] find:
   a) \((D \cup E) \cap F\), b) \((D \cap F)’\)
5. **Develop set laws by inductive or deductive reasoning.**
   
i) Use previously established set laws to simplify $A \cap (A' \cup B)$ What type of reasoning was used?
   
ii) Prove the associative law of intersection by deductive reasoning using Venn diagrams.
   
iii) If $A$ is a set that has $k$ elements, then $A$ has:
      a) $k^2$ subsets, b) $2^k$ subsets, c) $2^k - 1$ subsets, d) none of the above.
   
iv) DeMorgan’s Law for sets states $(A \cup B)' =$
      a) $A' \cup B'$, b) $A' \cap B'$, c) $A \cap B$, d) none of the above.

6. **Solve applied problems (e.g., survey analysis) using set operations.**
   
i) In a fraternity with 37 members, 18 take math, 5 take both math and chemistry, and 8 take neither math nor chemistry. How many take chemistry but not math?
   
ii) Thirty people were surveyed on what flavors of ice cream they like. 16 like vanilla, 21 like chocolate, 17 like strawberry, 8 like only vanilla and chocolate, 7 like vanilla and strawberry, 11 like chocolate and strawberry, and 5 like all three flavors. Using $V$ for like vanilla, $C$ for like chocolate, and $S$ for like strawberry, label all eight regions of the Venn diagram below with the number of people in each region.

![Venn Diagram](image)

7. **Perform operations of conjunction, disjunction, and negation on logic statements that are either represented symbolically or by Euler diagrams.**
   
i) Use Euler circles to draw a diagram for each of the following statements. Correctly label each circle using $p$ for “people”, and $h$ for “honest.” Show solid dots that indicate the relevant regions.
      a) “All people are honest.”
      b) “No people are honest.”
      c) “Some people are honest.”
      d) “Some people are not honest.”
   
ii) $p$ is T, $q$ is T, and $r$ is F. Determine whether the compound expressions is T or F:
      a) $(p \land r) \rightarrow q$
      b) $(p \lor r) \leftrightarrow (p \land q)$
8. Construct truth tables for compound logic statements.

i) Make a truth table for the compound statement \( \neg p \lor q \rightarrow (p \land \neg r) \). Circle the final column.

ii) Fill in each column with the correct truth values, using T or F.

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iii) Is \( p \rightarrow q \) logically equivalent to \( \neg p \lor q \)? Make a truth and circle the column you are comparing, then answer “yes” or “no”.

9. Develop logic laws by inductive or deductive reasoning.

i) Prove the other DeMorgan’s Law for Logic: \( \neg (p \land q) \) L.E. \( \neg p \lor \neg q \)

ii) Prove the other Associative Law for Logic: \( p \lor (q \lor r) \) L.E. \( (p \lor q) \lor r \)

iii) Prove the other Distributive Law for Logic: \( p \land (q \lor r) \) L.E. \( (p \land q) \lor (p \land r) \)

10. Determine the validity of logical arguments by truth tables, standard arguments (e.g., the Law of Contraposition, the Fallacy of the Inverse), or Euler circles.

i) Draw and label an Euler diagram which shows the argument is invalid. Use \( d \) for “dogs”, \( f \) for “friendly”, and \( c \) for “cats”. Show solid dots as appropriate:

   All dogs are friendly.
   Some cats are friendly.
   . . .No dogs are cats.

ii) Fill in the blanks to complete each standard form, both valid and invalid. \( LD = \) Law of Detachment, \( LC = \) Law of Contraposition, \( LS = \) Law of Syllogism, \( DS = \) Disjunctive Syllogism, \( FC = \) Fallacy of the Converse, and \( FI = \) Fallacy of the Inverse. The first premise of each argument is given.

<table>
<thead>
<tr>
<th>LD</th>
<th>LC</th>
<th>LS</th>
<th>DS</th>
<th>FC</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \rightarrow q )</td>
<td>( p \rightarrow q )</td>
<td>( p \rightarrow q )</td>
<td>( p \lor q )</td>
<td>( p \rightarrow q )</td>
<td>( p \rightarrow q )</td>
</tr>
<tr>
<td>( \therefore )</td>
<td>( \therefore )</td>
<td>( \therefore )</td>
<td>( \therefore )</td>
<td>( \therefore )</td>
<td>( \therefore )</td>
</tr>
</tbody>
</table>

iii) First write the argument symbolically. Choose the standard form that it matches, (LD, LC, LS, DS, FC, or FI), then state if the argument is “valid” or “invalid”.

   a. “If you study, then you will pass. You did not study. Therefore you did not pass.” Standard form ________ argument is ________

   b. “If the moon orbits the earth, then a cat is a dog. A cat is not a dog. Therefore the moon does not orbit the earth.” Standard form ________ argument is ________

   c. “If it rains, then I will not go to the beach. If I do not go to the beach, then I will study. Therefore if it rains, then I will study.” Standard form ________ argument is ________
11. Represent numbers using various numeration systems, both ancient and modern.

i) Find the value of \( CMXXVIDCCXLIX \).
M = 1000, D = 500, C = 100, L = 50, X = 10, V = 5, and I = 1.
Two vertical bars around a group = multiply the group by 100.
A horizontal bar above a group = multiply the group by 1000.

ii) Find the value of \( <<<|| \text{ <| } <<<|| \).
<= 100, | = 1, \( \text{<|>l} \) = subtract

iii) Convert 110102 to base ten.
iv) Convert 3214 to base ten.
v) Convert 3AD16 to base ten.
vi) Convert 20 to base three.
vii) Convert 192 to base five.
viii) Convert 928 to base sixteen.

12. Perform arithmetic operations in various numeration systems.

i) Use the Egyptian doubling procedure to multiply 19 x 25. Use base ten numerals, not Egyptian numerals. Show all work.

ii) Perform each computation. Show all steps, including carries and borrows, using only base two numerals. Base two addition and multiplication tables are provided for reference.

<table>
<thead>
<tr>
<th>Add</th>
<th>0</th>
<th>1</th>
<th>Multiply</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
<td></td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

a) 11101\text{\textsubscript{2}} + 1110\text{\textsubscript{2}}

b) 11101\text{\textsubscript{2}} - 1110\text{\textsubscript{2}}

c) 1011\text{\textsubscript{2}} \times 101\text{\textsubscript{2}}

d) 11101\text{\textsubscript{2}} divided by 11\text{\textsubscript{2}}

iii) Perform each computation. Show all steps, including carries and borrows, using only base five numerals. Base five addition and multiplication tables are provided for reference.

<table>
<thead>
<tr>
<th>Add</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Mult</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
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<td>11</td>
<td>12</td>
<td>13</td>
<td></td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>13</td>
<td>22</td>
</tr>
</tbody>
</table>

a) 31200\text{\textsubscript{5}} - 2331\text{\textsubscript{5}}

b) 23\text{\textsubscript{5}} \times 32\text{\textsubscript{5}}

13. Apply the binary, octal, and hexadecimal numeration systems to the modern digital world.

i) Convert 11010101\text{\textsubscript{2}} to base eight.

ii) Convert 3E7\text{\textsubscript{16}} to base two.

iii) Convert 327\text{\textsubscript{8}} to base sixteen.
14. Count the elements of finite sets by systematic listings, trees, permutations, combinations, or the Fundamental Counting Principle.

i) At a local restaurant, customers can order a complete meal consisting of three choices: main course, starch, and vegetables. The main course can be meat (m), chicken (c), or fish (f). If a customer chooses meat, then he can choose for his starch either potatoes (p), rice (r), or noodles (n). If a customer chooses chicken or fish, then he can choose for his starch either potatoes or rice. For vegetables, all complete meals come with a choice of string beans (s) or broccoli (b). Make a tree diagram, and then use it to count the total possible number of complete meals.

ii) A local bottled water company assigns each of its customers and identification code. Each code begins with a letter of the alphabet from A to Z, followed by four numerical digits, from 0 to 9, (for example: K5209). If repletion of numerical digits is allowed, then what is the total number of identification codes possible?

iii) In the following situations, the number of ways to perform each task can be counted by using either a Permutation $P(n, r)$ or a Combination $C(n, r)$. State which method applies and specify the values of $n$ and $r$. Write answer in the form $P(n, r)$ or $C(n, r)$.

a) From a group of 30 players, an all-star team of 12 players will be formed.

b) A club of 25 members is going to elect a president, a vice president, and a treasurer.

c) From a deck of 52 playing cards, a hand of 5 cards will be dealt.

d) 9 people are to be arranged in a straight line.

iv) Solve by any appropriate method and show all work.

a) On a geography test, 7 states must be matched with 7 state capitals. What is the total number of possible ways to match states with capitals, (either correctly or incorrectly)?

b) Three coins are randomly tossed and the outcome is recorded, (for example, one possible outcome is HHT). What is the total number of possible outcomes?

15. Develop counting formulas by inductive or deductive reasoning.

i) Consider arranging people in a straight line: Allen, Bob, Carol, Dave, and Ellen. Repetitions are not allowed and order is important. But we will not arrange all 5 people in line; we will take only 3 of them. How many arrangements are possible?

a) Show a 3-stage tree to determine the number of arrangements.

b) Use the fundamental counting principle to determine the number of arrangements

c) Can you use factorials to get the same result? Show how.

Congratulations, you have just derived the formula for permutations $P(n, r) = \frac{n!}{(n-r)!}$

ii) What if order was not important; ABC was the same as CBA.

a) How many arrangements of 3 letters can be made from {A, B, C, D}; use a 3-stage tree.

b) Assume order is not important and remove the repetitions and state how many arrangements of 3 letters remain.

iii) Note that $C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r!(n-r)!}$

i) Express each probability as a fraction reduced to lowest terms.
   a. A single card will be randomly drawn from a standard deck of 52 playing cards. Find the probability of drawing a face card.
   b. Three fair coins are randomly tossed. (HTT is one possible outcome). Find the probability of obtaining exactly two heads.
   c. A pair of fair dice is rolled once. For example, (2, 6) is one possible outcome. What is the probability of obtaining a total of seven?
   d. Ten balls are placed in a jar: 5 red, 3 blue, and 2 green. A person randomly selects two balls from the jar, without replacement. What is the probability the neither ball is blue?

17. Determine expected values of real-life events in games.

i) The probability that a 80-year old male in the United States will die within one year is 0.073. An insurance company will offer a one-year policy to such a person for $500, non-refundable. If the person dies within one year, the beneficiaries will receive $5,000. What is the person’s expected value on one policy?

ii) A church has sold 500 raffle tickets for $5 each, non-refundable. One first prize of $500 will be awarded, along with two second prizes of $250 and two third place prizes of $100. If you purchase one ticket, what is your expected value? Round to the nearest penny.

18. Summarize real-life data sets by bar graphs or histograms and interpret such summaries.

i) The bar graph summarizes a group of students who have taken a test.

```
<table>
<thead>
<tr>
<th>Test Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-30</td>
<td>1</td>
</tr>
<tr>
<td>31-40</td>
<td>2</td>
</tr>
<tr>
<td>41-50</td>
<td>1</td>
</tr>
<tr>
<td>51-60</td>
<td>4</td>
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<td>61-70</td>
<td>5</td>
</tr>
<tr>
<td>71-80</td>
<td>6</td>
</tr>
<tr>
<td>81-90</td>
<td>3</td>
</tr>
<tr>
<td>91-100</td>
<td>1</td>
</tr>
</tbody>
</table>
```

a) How many students took the test?
b) How many students achieved a score of at least 71?
c) What range of scores occurred most frequently?
ii) The weights (in pounds) of students in a class were distributed as follows: 115, 61, 75, 85, 79, 101, 89, 81, 99, and 90. Make a histogram that begins at 60 pounds and uses a class width of 20 pounds. Be sure to label the class width boundaries on the Pounds axis.

19. Determine means, medians, modes, ranges, and standard deviations of real-life data sets.

i) For each data set given, calculate the three ways to measure central tendency.
   a) 1, 1, 1, 3, 5, 14
   b) 3, 3, 3, 6, 9, 9, 9
   c) \[ \begin{array}{c|c} x & f \\ \hline 2 & 3 \\ 3 & 1 \\ 5 & 2 \\ 6 & 5 \\ 8 & 3 \\ 11 & 1 \end{array} \]

ii) The students in a class received the following scores: 80, 76, 81, 84, 79, 80, 90, 75, 75, and 80.

   a) Calculate the mean and standard deviation of these scores; round to the nearest tenth. Show all steps in the calculation.

iii) The following scores were achieved by a group of golfers on the first day of a tournament. Calculate the standard deviation of the scores, rounded to the nearest tenth. Show all steps.

<table>
<thead>
<tr>
<th>Score</th>
<th>72</th>
<th>73</th>
<th>78</th>
<th>84</th>
<th>86</th>
<th>93</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

20. Solve applied problems (e.g., grade determination, stock volatility) by using descriptive statistics.

i) The students in a class received the following scores: 80, 76, 81, 84, 79, 80, 90, 75, 75, and 80.

   a) Calculate the mean and standard deviation of these scores; round to the nearest tenth. Show all steps in the calculation.
   b) Calculate how many standard deviations from the mean, a score of 76 is and round to the nearest tenth.
   c) Determine what letter grade corresponds to a grade of 76.
ii) You are given information on the performance of two stocks over a one-month period. The stock with a greater coefficient of variation is considered more volatile. Calculate the coefficient of variation for both stocks, rounded to the nearest tenth, and state which is more volatile.

Safeway: mean daily closing price = $156.23 with standard deviation = $15.50
Foodland: mean daily closing price = $94.86 with standard deviation = $11.40